

1 Winkelfunktionen

r	5	1,22	1
x	4	0,7071	0
y	3	1	1
α	36,86°	54,7°	90°

1. $\alpha = \arcsin(\frac{y}{r}) = \arcsin(\frac{3}{5}) = 36,86^\circ$
2. $x = \cos(\alpha) * r = \cos(36,86^\circ) * 5 = 4$
3. $\alpha = \arctan(\frac{y}{x}) = \arctan(\frac{1}{0,7071}) = 54,7^\circ$
4. $r = \frac{y}{\sin(\alpha)} = \frac{1}{\sin(54,7^\circ)} = 1,22$
5. $y = \sin(\alpha) * r = \sin(90^\circ) * 1 = 1$
6. $x = \cos(\alpha) * r = \cos(90^\circ) * 1 = 0$

2 Komplexe Zahlen

a)

$$\underline{A} = 2 * \cos(120^\circ) + j2 * \sin(120^\circ)$$

$$\underline{D} = 1$$

$$\underline{A} = -1 + j1,732$$

$$\underline{D} = 1 * \cos(0^\circ) + j \sin(0^\circ)$$

$$\underline{A} = \sqrt{(-1)^2 + 1,732^2} = 2 * e^{j120^\circ}$$

$$\underline{D} = 1 * e^{j0^\circ}$$

$$\underline{B} = 4,13 * e^{j76^\circ}$$

$$\underline{E} = 0,51 * e^{-j78,7^\circ}$$

$$\underline{B} = 4,13 * \cos(76^\circ) + j4,13 * \sin(76^\circ)$$

$$\underline{E} = 0,51 * \cos(-78,7^\circ) + j * 0,51 * \sin(-78,7^\circ)$$

$$\underline{B} = 1 + j4$$

$$\underline{E} = 0,1 - j0,5$$

$$\underline{C} = -j$$

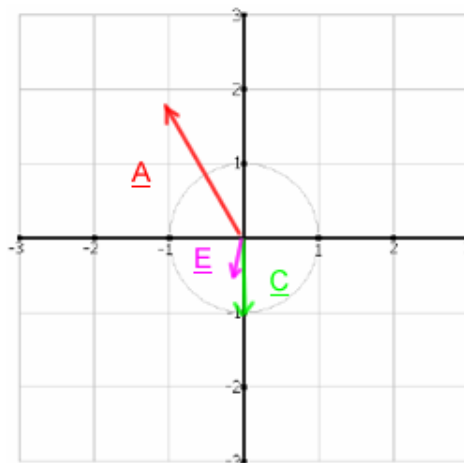
$$\underline{F} = 1,73 + j$$

$$\underline{C} = 1 * \cos(-90^\circ) + j \sin(-90^\circ)$$

$$\underline{F} = 2 * \cos(30,03^\circ) + j * 2 \sin(30,03^\circ)$$

$$\underline{C} = 1 * e^{-j90^\circ}$$

$$\underline{F} = 2 * e^{j30,03^\circ}$$



b)

$$\underline{F} - \underline{D} = 1,73 + j - 1$$

$$\underline{F} - \underline{D} = 0,73 + j$$

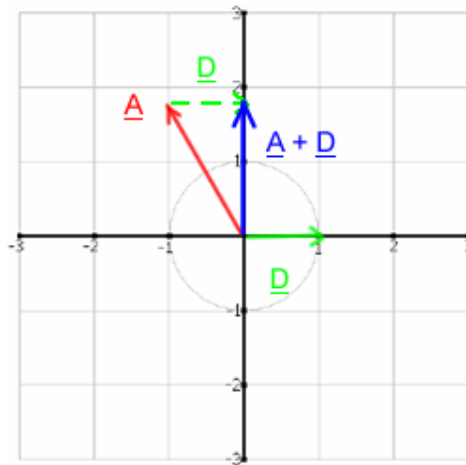
$$\underline{A} + \underline{D} = -1 + j1,732 + 1$$

$$\underline{A} + \underline{D} = j1,732$$

$$\frac{\underline{A}}{\underline{E}} = \frac{2 * e^{j120^\circ}}{0,51 * e^{-j78,7^\circ}}$$

$$\frac{\underline{A}}{\underline{E}} = \frac{2}{0,51} * e^{j120^\circ + j78,7^\circ}$$

$$\frac{\underline{A}}{\underline{E}} = 3,92 * e^{-j161,3^\circ}$$



3 Impedanz

Gegeben:

$$\underline{U}_{AB} = 10e^{-j60^\circ} V$$

$$\underline{Z}_1 = 10e^{j0^\circ} \Omega$$

$$\underline{Z}_2 = 10e^{-j90^\circ} k\Omega$$

$$\underline{Z}_3 = 20e^{j45^\circ} k\Omega$$

$$\underline{Z}_4 = 10e^{j90^\circ} k\Omega$$

$$\underline{Z}_5 = 0e^{j0^\circ} k\Omega$$

Gesucht: \underline{Z}_{AB} , alle \underline{I} , alle \underline{U} , Phasenlage \underline{I} zu \underline{U}_{AB}

Lösung:

1. Berechnung Parallelschaltung

$$\underline{Z}_{234} = \frac{\underline{Z}_2 \underline{Z}_3 + \underline{Z}_2 \underline{Z}_4 + \underline{Z}_3 \underline{Z}_4}{\underline{Z}_2 \underline{Z}_3 \underline{Z}_4}$$

$$\underline{Z}_{234} = \frac{200e^{-j45^\circ} k\Omega + 100e^{j0^\circ} k\Omega + 200e^{j135^\circ} k\Omega}{2000e^{j45^\circ}}$$

$$\underline{Z}_{234} = \frac{141,42 - j141,42 k\Omega + 100 k\Omega - 141,42 + j141,42 k\Omega}{2000e^{j45^\circ}}$$

$$\underline{Z}_{234} = \frac{100 k\Omega}{2000e^{j45^\circ}}$$

$$\underline{Z}_{234} = \frac{100e^{j0^\circ} k\Omega}{2000e^{j45^\circ}}$$

$$\underline{Z}_{234} = 0,05e^{-j45^\circ} k\Omega$$

2. Berechnung Gesamtimpedanz

$$\underline{Z}_{AB} = \underline{Z}_1 + \underline{Z}_{234} + \underline{Z}_5$$

$$\underline{Z}_{AB} = 10\Omega + 35,36 - j35,36\Omega + 0\Omega$$

$$\underline{Z}_{AB} = 45,36 - j35,36\Omega$$

$$\underline{Z}_{AB} = 57,51e^{-j37,94^\circ}\Omega$$

Die Gesamtimpedanz beträgt $58e^{-j38^\circ}\Omega$.

3. Berechnung der Spannungen und Ströme

$$\underline{U}_{AB} = \underline{U}_1 + \underline{U}_2 + \underline{U}_5$$

$$\underline{U}_2 = \underline{U}_3 = \underline{U}_4$$

$$\underline{I}_{AB} = \underline{I}_1 = \underline{I}_5$$

$$\underline{I}_{AB} = \underline{I}_2 + \underline{I}_3 + \underline{I}_4$$

$$\underline{I}_{AB} = \frac{\underline{U}_{AB}}{\underline{Z}_{AB}}$$

$$\underline{I}_{AB} = \frac{10e^{-j60^\circ} V}{57,51e^{-j37,94^\circ} \Omega}$$

$$\underline{I}_{AB} = 0,17e^{-j22,06^\circ} A$$

$$\underline{U}_1 = \underline{Z}_1 * \underline{I}_1$$

$$\underline{U}_1 = 10e^{j0^\circ} \Omega * 0,17e^{-j22,06^\circ} A$$

$$\underline{U}_1 = 1,7e^{-j22,06^\circ} V$$

$$\underline{U}_5 = \underline{Z}_5 * \underline{I}_5$$

$$\underline{U}_5 = 0e^{j0^\circ} \Omega * 0,17e^{-j22,06^\circ} A$$

$$\underline{U}_5 = 0e^{-j22,06^\circ} V$$

$$\underline{U}_2 = \underline{U}_{AB} - \underline{U}_1 - \underline{U}_5$$

$$\underline{U}_2 = 5 - j8,66V - (1,58 - j0,64V) - 0V$$

$$\underline{U}_2 = 3,42 - j8,02V$$

$$\underline{U}_2 = 8,72e^{-j66,9^\circ} V$$

$$\underline{I}_2 = \frac{\underline{U}_2}{\underline{Z}_2}$$

$$\underline{I}_2 = \frac{8,72e^{-j66,9^\circ} V}{10e^{-j90^\circ} k\Omega}$$

$$\underline{I}_2 = 0,87e^{j23,1^\circ} mA$$

$$\underline{I}_3 = \frac{\underline{U}_3}{\underline{Z}_3}$$

$$\underline{I}_3 = \frac{8,72e^{-j66,9^\circ} V}{20e^{j45^\circ} k\Omega}$$

$$\underline{I}_3 = 0,44e^{-j111,9^\circ} mA$$

$$\underline{I}_4 = \frac{\underline{U}_4}{\underline{Z}_4}$$

$$\underline{I}_4 = \frac{8,72e^{-j66,9^\circ} V}{10e^{j90^\circ} k\Omega}$$

$$\underline{I}_4 = 0,87e^{-j156,9^\circ} mA$$

Der Gesamtstrom eilt der Gesamtspannung um knapp 38° voraus.

4 RLC-Schaltung

Siehe Tabelle 1.

5 RC-Schaltung

Gegeben: Reihenschaltung aus $R = 400\Omega$ und Kondensator $\underline{Z} = 500e^{-j36,4^\circ}\Omega$.

$I = 2mA = 0,002A$, $T = 16,7ms = 0,0167s$

Gesucht: C , \underline{U}

Lösung:

$$Z = \sqrt{R^2 + X_C^2}$$

$$X_C = \sqrt{Z^2 - R^2}$$

$$X_C = \sqrt{(500\Omega)^2 - (400\Omega)^2}$$

$$X_C = 300\Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$f = \frac{1}{T}$$

$$C = \frac{1}{2\pi f X_C}$$

$$f = 59,88Hz$$

$$C = \frac{1}{2\pi * 59,88Hz * 300\Omega}$$

$$C = 8,86\mu F$$

$$U = I * Z$$

$$U = 0,002A * 500\Omega$$

$$U = 1V$$

$$\phi = \arctan \frac{-X_C}{R}$$

$$\phi = \arctan \frac{-300\Omega}{400\Omega}$$

$$\phi = -36,87^\circ$$

$$\underline{U} = 1e^{-j36,78^\circ}$$

Die Kapazität des Kondensators beträgt $8,7\mu F$. Die Spannung über der Schaltung beträgt $1V$ und die Spannung hat eine Phasenverschiebung von $-36,8^\circ$ gegenüber dem Strom. Der Strom eilt also voraus.

6 Filter

Gegeben: Tiefpassfilter mit $R = 160\Omega$, $C = 100nF$

Gesucht: f_{Gr}

Lösung:

$$X_C = R$$

$$X_C = \frac{1}{2\pi fC}$$

$$f_{Gr} = \frac{1}{2\pi RC}$$

$$f_{Gr} = \frac{1}{2\pi 160\Omega * 100 * 10^{-9}F}$$

$$f_{Gr} = 9,95kHz$$

Der Rest siehe Tabelle 2.