Theoretische Physik für LAK II: Übung 1

von
Naja v. Schmude (4127652), Gruppe: II

Aufgabe 1

Zunächst die Skizzen der Vektorfelder ...

Nun berechnen wir die Divergenz.

$$\vec{\nabla}(\alpha(-y, x-a, 0)) = \frac{\partial(-\alpha y)}{\partial x} + \frac{\partial \alpha(x-a)}{\partial y} + \frac{\partial 0}{\partial z} = 0$$

$$\vec{\nabla}\vec{e_r} = \frac{1}{r^2}\frac{\partial(r^2 * 1)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta * 0)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial 0}{\partial\phi} = \frac{2}{r}$$

$$\vec{\nabla}(x^2, y^2, z^2) = \frac{\partial x^2}{\partial x} + \frac{\partial y^2}{\partial y} + \frac{\partial z^2}{\partial z} = 2x + 2y + 2z$$

$$\vec{\nabla}\alpha\vec{e_\theta} = \frac{1}{r^2}\frac{\partial(r^2 * 0)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta * 1)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial 0}{\partial\phi} = \frac{\cos\theta}{r\sin\theta}$$

Und nun zur Rotation.

$$\vec{\nabla}\times(\alpha(-y, x-a, 0)) = \vec{e_x}\left(\frac{\partial 0}{\partial y} - \frac{\partial\alpha(x-a)}{\partial z}\right) + \vec{e_y}\left(\frac{\partial - \alpha y}{\partial z} - \frac{\partial 0}{\partial x}\right) + \vec{e_z}\left(\frac{\partial\alpha(x-a)}{\partial x} - \frac{\partial - \alpha y}{\partial y}\right)$$
$$= (0, 0, 2\alpha)$$

$$\vec{\nabla}\times\vec{e_r} = \vec{e_r}\frac{1}{r\sin\theta}\left(\frac{\partial\sin\theta 0}{\partial\theta} - \frac{\partial 0}{\partial\phi}\right) + \vec{e_\theta}\left(\frac{1}{r\sin\theta}\frac{\partial 1}{\partial\phi} - \frac{1}{r}\frac{\partial r * 0}{\partial r}\right) + \vec{e_\phi}\left(\frac{\partial r * 0}{\partial r} - \frac{\partial 1}{\partial\theta}\right)$$
$$= (0, 0, 0)$$

$$\vec{\nabla}\times(x^2, y^2, z^2) = \vec{e_x}\left(\frac{\partial z^2}{\partial y} - \frac{\partial y^2}{\partial z}\right) + \vec{e_y}\left(\frac{\partial x^2}{\partial z} - \frac{\partial z^2}{\partial x}\right) + \vec{e_z}\left(\frac{\partial y^2}{\partial x} - \frac{\partial x^2}{\partial y}\right)$$
$$= (0, 0, 0)$$

$$\vec{\nabla}\times\alpha\vec{e_\theta} = \vec{e_r}\frac{1}{r\sin\theta}\left(\frac{\partial\sin\theta 0}{\partial\theta} - \frac{\partial 1}{\partial\phi}\right) + \vec{e_\theta}\left(\frac{1}{r\sin\theta}\frac{\partial 0}{\partial\phi} - \frac{1}{r}\frac{\partial r * 0}{\partial r}\right) + \vec{e_\phi}\left(\frac{\partial r * 1}{\partial r} - \frac{\partial 0}{\partial\theta}\right)$$
$$= (0, 0, 1)$$

Aufgabe 2

Die Gesamtladung Q einer Kugel des Radius R und eines Würfels der Seitenlänge a mit einer Ladungsdichte von $\rho(r,\theta,\phi) = \rho_0 \frac{r^2}{r_0^2}$ sei zu berechnen.

a)

$$
\begin{aligned}
Q &= \int_{r=0}^{R} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \rho_0 \frac{r^2}{r_0^2} r^2 \sin\theta dr d\theta d\phi \\
&= \frac{\rho_0}{r_0^2} \int_0^R r^4 dr \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \\
&= \frac{\rho_0}{r_0^2} \left[\frac{1}{5} r^5\right]_0^R * [\phi]_0^{2\pi} * \underbrace{[-\cos\phi]_0^{\pi}}_{=0} \\
&= 0
\end{aligned}
$$

b) In kartesischen Koordinaten ergibt sich für die Ladungsdichte $\rho(x,y,z) = \rho_0 \frac{x^2+y^2+z^2}{r_0^2}$.

$$
\begin{aligned}
Q &= \int_{x=-a/2}^{a/2} \int_{y=-a/2}^{a/2} \int_{z=-a/2}^{a/2} \rho_0 \frac{x^2+y^2+z^2}{r_0^2} dx dy dz \\
&= \frac{\rho_0}{r_0^2} \int_{-a/2}^{a/2} x^2 dx \int_{-a/2}^{a/2} y^2 dy \int_{-a/2}^{a/2} z^2 dz \\
&= \frac{\rho_0}{r_0^2} \left[\frac{1}{3} x^3\right]_{-a/2}^{a/2} * \left[\frac{1}{3} y^3\right]_{-a/2}^{a/2} * \left[\frac{1}{3} z^3\right]_{-a/2}^{a/2} \\
&= 0
\end{aligned}
$$