Aufgabe 7

a) Berechne Würfelmittelpunkt

$$v_M = \frac{1}{8}(v_0 + \ldots + v_7) = \begin{pmatrix} 1/2 \\ 1/2 \\ 3/2 \end{pmatrix}$$

Transliere v_M in den Ursprung durch

$$T^{-1} := \begin{pmatrix} 1 & 0 & 0 & -1/2 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & -3/2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad , \text{ also ist } T = \begin{pmatrix} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Rotiere nun um $\pi/3$ um die x-Achse mittels

$$R_x\left(\frac{\pi}{3}\right) := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} & 0 \\ 0 & \sin\frac{\pi}{3} & \cos\frac{\pi}{3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 & 0 \\ 0 & \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Projiziere auf die xy-Ebene durch

$$P_{xy} := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b)

$$\vartheta = P_{xy} \cdot T \cdot R_x\left(\frac{\pi}{3}\right) \cdot T^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & -\frac{\sqrt{3}}{2} & \frac{1+3\sqrt{3}}{4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

c) Setze $\hat{v}_i := (v_i, 1)^T \quad , i = 0, \ldots, 7$

Es ergibt sich

$$\tilde{v}_0 := \vartheta \cdot \hat{v}_0 = \left(0, \frac{1+\sqrt{3}}{4}, 0, 1\right)^T$$
$$\tilde{v}_1 := \vartheta \cdot \hat{v}_1 = \left(1, \frac{1+\sqrt{3}}{4}, 0, 1\right)^T$$
$$\tilde{v}_2 := \vartheta \cdot \hat{v}_2 = \left(0, \frac{3+\sqrt{3}}{4}, 0, 1\right)^T$$
$$\tilde{v}_3 := \vartheta \cdot \hat{v}_3 = \left(1, \frac{3+\sqrt{3}}{4}, 0, 1\right)^T$$
$$\tilde{v}_4 := \vartheta \cdot \hat{v}_4 = \left(0, \frac{1-\sqrt{3}}{4}, 0, 1\right)^T$$
$$\tilde{v}_5 := \vartheta \cdot \hat{v}_5 = \left(1, \frac{1-\sqrt{3}}{4}, 0, 1\right)^T$$
$$\tilde{v}_6 := \vartheta \cdot \hat{v}_6 = \left(0, \frac{3-\sqrt{3}}{4}, 0, 1\right)^T$$
$$\tilde{v}_7 := \vartheta \cdot \hat{v}_7 = \left(1, \frac{3-\sqrt{3}}{4}, 0, 1\right)^T$$

d)