Aufgabe 8

a) $A = \begin{pmatrix} 3 & 3 & 0 \\ 3 & 5 & -2 \\ 0 & -2 & 2 \end{pmatrix}$ $\rightsquigarrow \det A = 3\cdot5\cdot2 + 3\cdot(-2)\cdot0 + 0\cdot3\cdot(-2)$

$-3\cdot(-2)\cdot(-2) - 3\cdot3\cdot2 - 0\cdot5\cdot0$

$= 30 - 0 + 0$

$- 12 - 18 - 0$

$= 0 \Rightarrow A$ singulär

b) $(v_1, v_2, v_3, 1)\begin{pmatrix} 3 & 3 & 0 & 6 \\ 3 & 5 & -2 & -8 \\ 0 & -2 & 2 & 2 \\ 6 & -8 & 2 & 14 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ 1 \end{pmatrix} = (3v_1+3v_2+6, 3v_1+5v_2-2v_3-8, -2v_2+2v_3+2, 6v_1-8v_2+2v_3+14)\begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ 1 \end{pmatrix}$

$= 3v_1^2 + 3v_1v_2 + 6v_1 + 3v_1v_2 + 5v_2^2 - 2v_2v_3 - 8v_2 - 2v_2v_3 + 2v_3^2 + 2v_3 + 6v_1 - 8v_2 + 2v_3 + 14$

$= 3v_1^2 + 6v_1v_2 + 12v_1 + 5v_2^2 - 4v_2\underbrace{v_3}_{=0} - 16v_2 + 2\underbrace{v_3^2}_{=0} + 4\underbrace{v_3}_{=0} + 14$

$v(\lambda) = (1-\lambda)p_1 + \lambda p_2$

$\overset{!}{=} 3(1\cdot(1-\lambda) + 3\cdot\lambda)^2 + 6(1\cdot(1-\lambda)+3\cdot\lambda)(-1\cdot(1-\lambda)+1\cdot\lambda) + 12\cdot[1(1-\lambda)+3\cdot\lambda]$

$+ 5(-1(1-\lambda)+1\cdot\lambda)^2 - 16(-1(1-\lambda)+1\cdot\lambda) + 14$

$= 3(2\lambda+1)^2 + 6(1+2\lambda)(2\lambda-1) + 12(1+2\lambda)$

$+ 5(2\lambda-1)^2 - 16(2\lambda-1) + 14$

$= 3(4\lambda^2+4\lambda+1) + 6(4\lambda^2-1) + 12(1+2\lambda) + 5\cdot(4\lambda^2-4\lambda+1) - 16(2\lambda-1) + 14$

$= 12\lambda^2 + 12\lambda + 3 + 24\lambda^2 - 6 + 12 + 24\lambda + 20\lambda^2 - 20\lambda + 5 - 32\lambda + 16 + 14$

$= 56\lambda^2 - 16\lambda + 44$

Finde Minimum

$\frac{\partial}{\partial\lambda} v(\lambda)^T Q\, v(\lambda) = 112\lambda - 16 \overset{!}{=} 0 \Leftrightarrow \lambda = \frac{16}{112} = \frac{1}{7}$

$\Rightarrow v^* = v\left(\frac{1}{7}\right) = \left(1-\frac{1}{7}\right)\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + \frac{1}{7}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6/7 + 3/7 \\ -6/7 + 1/7 \\ 0 + 0 \end{pmatrix} = \begin{pmatrix} 9/7 \\ -5/7 \\ 0 \end{pmatrix}$