Aufgabe 18

1) $P(t) = \sum_{i=0}^{n} p_i \prod_{k=0,\, k\neq i}^{n} \frac{t - t_k}{t_i - t_k}$

Hier: Nur 3 Polygonenzüge:

$$P(t) = \sum_{i=0}^{3} p_i \prod_{k=0,\, k\neq i}^{3} \frac{t-t_k}{t_i - t_k} = A\left(\frac{t-t_1}{t_0-t_1} \cdot \frac{t-t_2}{t_0-t_2}\right) + B\left(\frac{t-t_0}{t_1-t_0} \cdot \frac{t-t_2}{t_1-t_2}\right) + C\left(\frac{t-t_0}{t_2-t_0} \cdot \frac{t-t_1}{t_2-t_1}\right)$$

$$= A \cdot \frac{t^2 - t(t_1+t_2) + t_1 t_2}{t_0^2 - t_0(t_1+t_2) + t_1 t_2} + B \cdot \frac{t^2 - t(t_0+t_2) + t_0 t_2}{t_1^2 - t_1(t_0+t_2) + t_0 t_2} + C \frac{t^2 - t(t_0+t_1) + t_0 t_1}{t_2^2 - t_2(t_0+t_1) + t_0 t_1}$$

Mit $P(0) = A$, $P(\frac{1}{2}) = B$ und $P(1) = C$ folgt unmittelbar

$t_0 = 0$, $t_1 = \frac{1}{2}$ und $t_2 = 1$.

Also

$$P(t) = A \cdot \frac{t^2 - \frac{3}{2}t + \frac{1}{2}}{\frac{1}{2}} + B \frac{t^2 - t}{\frac{1}{4} - \frac{1}{2}} + C \frac{t^2 - \frac{1}{2}t}{1 - \frac{1}{2}}$$

$$\underline{\underline{= (2t^2 - 3t + 1)A - 4(t^2 - t)B + (2t^2 - t)C}}$$

)

$$P\left(\tfrac{1}{4}\right) = \left(\tfrac{2}{16} - \tfrac{3}{4} + 1\right)A - 4\left(\tfrac{1}{16} - \tfrac{1}{4}\right)B + \left(\tfrac{2}{16} - \tfrac{1}{4}\right)C = \tfrac{3}{8}A + \tfrac{3}{4}B - \tfrac{1}{8}C$$

$$= \left[\tfrac{3}{8}\begin{pmatrix}1\\3\end{pmatrix} + \tfrac{3}{4}\begin{pmatrix}1\\2\end{pmatrix} - \tfrac{1}{8}\begin{pmatrix}0\\2\end{pmatrix},\ \tfrac{3}{8}\begin{pmatrix}1\\0\end{pmatrix} + \tfrac{3}{4}\begin{pmatrix}1\\1\end{pmatrix} - \tfrac{1}{8}\begin{pmatrix}0\\1\end{pmatrix},\ \tfrac{3}{8}\begin{pmatrix}2\\0\end{pmatrix} + \tfrac{3}{4}\begin{pmatrix}2\\1\end{pmatrix} - \tfrac{1}{8}\begin{pmatrix}3\\1\end{pmatrix},\ \tfrac{3}{8}\begin{pmatrix}2\\3\end{pmatrix} + \tfrac{3}{4}\begin{pmatrix}2\\2\end{pmatrix} - \tfrac{1}{8}\begin{pmatrix}3\\2\end{pmatrix}\right]$$

$$= \left[\begin{pmatrix}9/8\\19/8\end{pmatrix}, \begin{pmatrix}9/8\\5/8\end{pmatrix}, \begin{pmatrix}15/8\\5/8\end{pmatrix}, \begin{pmatrix}15/8\\19/8\end{pmatrix}\right]$$

$$P\left(\tfrac{3}{4}\right) = \left(\tfrac{18}{16} - \tfrac{9}{4} + 1\right)A - 4\left(\tfrac{9}{16} - \tfrac{3}{4}\right)B + \left(\tfrac{18}{16} - \tfrac{3}{4}\right)C = -\tfrac{1}{8}A + \tfrac{3}{4}B + \tfrac{3}{8}C$$

$$= \left[-\tfrac{1}{8}\begin{pmatrix}1\\3\end{pmatrix} + \tfrac{3}{4}\begin{pmatrix}1\\2\end{pmatrix} + \tfrac{3}{8}\begin{pmatrix}0\\2\end{pmatrix},\ -\tfrac{1}{8}\begin{pmatrix}1\\0\end{pmatrix} + \tfrac{3}{4}\begin{pmatrix}1\\1\end{pmatrix} + \tfrac{3}{8}\begin{pmatrix}0\\1\end{pmatrix},\ -\tfrac{1}{8}\begin{pmatrix}2\\0\end{pmatrix} + \tfrac{3}{4}\begin{pmatrix}2\\1\end{pmatrix} + \tfrac{3}{8}\begin{pmatrix}3\\1\end{pmatrix},\ -\tfrac{1}{8}\begin{pmatrix}2\\3\end{pmatrix} + \tfrac{3}{4}\begin{pmatrix}2\\2\end{pmatrix} + \tfrac{3}{8}\begin{pmatrix}3\\2\end{pmatrix}\right]$$

$$= \left[\begin{pmatrix}5/8\\15/8\end{pmatrix}, \begin{pmatrix}5/8\\9/8\end{pmatrix}, \begin{pmatrix}19/8\\9/8\end{pmatrix}, \begin{pmatrix}19/8\\15/8\end{pmatrix}\right]$$