Tutor: Max

23.1.11

Scientific Visualization
Übung 9

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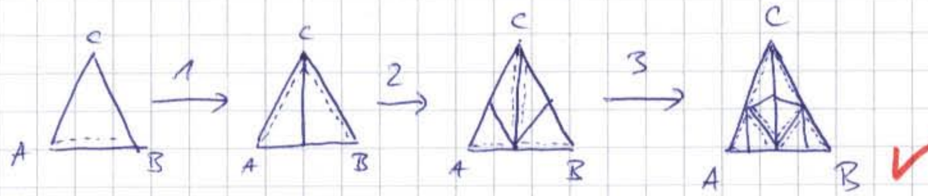
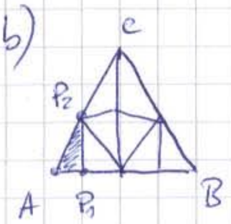
10/10

17. a) 3-Rivara-Schritte



b)



Gesucht Schwerpunkt S

$$S = \frac{1}{3}A + \frac{1}{3}P_1 + \frac{1}{3}P_2$$

$$P_1 = \frac{1}{2}A + \frac{1}{2}\left(\frac{1}{2}A + \frac{1}{2}B\right) = \frac{3}{4}A + \frac{1}{4}B$$

$$P_2 = \frac{1}{2}A + \frac{1}{2}C$$

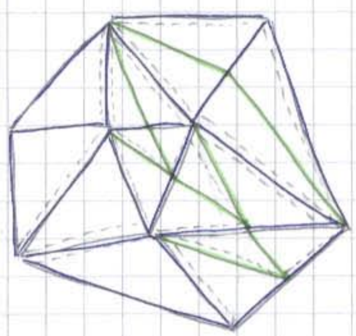
$$S = \frac{1}{3}A + \frac{1}{3}\left(\frac{3}{4}A + \frac{1}{4}B\right) + \frac{1}{3}\left(\frac{1}{2}A + \frac{1}{2}C\right)$$

$$= \frac{1}{3}A + \frac{1}{4}A + \frac{1}{12}B + \frac{1}{6}A + \frac{1}{6}C$$

$$= \frac{3}{4}A + \frac{1}{12}B + \frac{1}{6}C$$

Baryzentrische Koordinaten: $(3/4, 1/12, 1/6)$

c) 1-Rivara-Schritt



— alte Kanten
— neue Kanten
--- Verfeinerungskanten

Aufgabe 18

a) $P(t) = \sum_{i=0}^{n} p_i \prod_{\substack{k=0,\\ k\neq i}}^{n} \frac{t-t_k}{t_i-t_k}$

Hier: Nur 3 Polygonenzüge:

$$P(t) = \sum_{i=0}^{3} p_i \prod_{\substack{k=0,\\ k\neq i}}^{3} \frac{t-t_k}{t_i-t_k} = A\left(\frac{t-t_1}{t_0-t_1}\cdot\frac{t-t_2}{t_0-t_2}\right) + B\left(\frac{t-t_0}{t_1-t_0}\cdot\frac{t-t_2}{t_1-t_2}\right) + C\left(\frac{t-t_0}{t_2-t_0}\cdot\frac{t-t_1}{t_2-t_1}\right)$$

$$= A\cdot\frac{t^2-t(t_1+t_2)+t_1t_2}{t_0^2-t_0(t_1+t_2)+t_1t_2} + B\cdot\frac{t^2-t(t_0+t_2)+t_0t_2}{t_1^2-t_1(t_0+t_2)+t_0t_2} + C\,\frac{t^2-t(t_0+t_1)+t_0t_1}{t_2^2-t_2(t_0+t_1)+t_0t_1}$$

Mit $P(0)=A$, $P(\frac{1}{2})=B$ und $P(1)=C$ folgt unmittelbar

$t_0=0$, $t_1=\frac{1}{2}$ und $t_2=1$.

Also

$$P(t) = A\cdot\frac{t^2-\frac{3}{2}t+\frac{1}{2}}{\frac{1}{2}} + B\,\frac{t^2-t}{\frac{1}{4}-\frac{1}{2}} + C\,\frac{t^2-\frac{1}{2}t}{1-\frac{1}{2}}$$

$$\underline{\underline{=(2t^2-3t+1)A-4(t^2-t)B+(2t^2-t)C}}$$

b)

$$P\left(\tfrac{1}{4}\right) = \left(\tfrac{2}{16}-\tfrac{3}{4}+1\right)A-4\left(\tfrac{1}{16}-\tfrac{1}{4}\right)B+\left(\tfrac{2}{16}-\tfrac{1}{4}\right)C = \tfrac{3}{8}A+\tfrac{3}{4}B-\tfrac{1}{8}C$$

$$= \left[\tfrac{3}{8}\binom{1}{3}+\tfrac{3}{4}\binom{1}{2}-\tfrac{1}{8}\binom{0}{2},\ \tfrac{3}{8}\binom{1}{0}+\tfrac{3}{4}\binom{1}{1}-\tfrac{1}{8}\binom{0}{1},\ \tfrac{3}{8}\binom{2}{0}+\tfrac{3}{4}\binom{2}{1}-\tfrac{1}{8}\binom{3}{1},\ \tfrac{3}{8}\binom{2}{3}+\tfrac{3}{4}\binom{2}{2}-\tfrac{1}{8}\binom{3}{2}\right]$$

$$= \left[\binom{9/8}{19/8},\ \binom{9/8}{5/8},\ \binom{15/8}{5/8},\ \binom{15/8}{19/8}\right]$$

$$P\left(\tfrac{3}{4}\right) = \left(\tfrac{18}{16}-\tfrac{9}{4}+1\right)A-4\left(\tfrac{9}{16}-\tfrac{3}{4}\right)B+\left(\tfrac{18}{16}-\tfrac{3}{4}\right)C = -\tfrac{1}{8}A+\tfrac{3}{4}B+\tfrac{3}{8}C$$

$$= \left[-\tfrac{1}{8}\binom{1}{3}+\tfrac{3}{4}\binom{1}{2}+\tfrac{3}{8}\binom{0}{2},\ -\tfrac{1}{8}\binom{1}{0}+\tfrac{3}{4}\binom{1}{1}+\tfrac{3}{8}\binom{0}{1},\ -\tfrac{1}{8}\binom{2}{0}+\tfrac{3}{4}\binom{2}{1}+\tfrac{3}{8}\binom{3}{1},\ -\tfrac{1}{8}\binom{2}{3}+\tfrac{3}{4}\binom{2}{2}+\tfrac{3}{8}\binom{3}{2}\right]$$

$$= \left[\binom{5/8}{15/8},\ \binom{5/8}{9/8},\ \binom{19/8}{9/8},\ \binom{19/8}{15/8}\right]$$

c)